Reinforcement Learning-based Knowledge Distillation with LLM-as-a-Judge

Yiyang Shen[1] **Lifu Tu**[2*] **Weiran Wang**[1*]
[1]Department of Computer Science, University of Iowa
{yiyang-shen,weiran-wang}@uiowa.edu
[2]lifutu8@gmail.com

Abstract

Reinforcement Learning (RL) has been shown to substantially improve the reasoning capability of small and large language models (LLMs), but existing approaches typically rely on verifiable rewards, hence ground truth labels. We propose an RL framework that uses rewards from an LLM that acts as a judge evaluating model outputs over large amounts of unlabeled data, enabling label-free knowledge distillation and replacing the need of ground truth supervision. Notably, the judge operates with a single-token output, making reward computation efficient. When combined with verifiable rewards, our approach yields substantial performance gains across math reasoning benchmarks. These results suggest that LLM-based evaluators can produce effective training signals for RL fine-tuning.

1 Introduction

Recent advancements demonstrate that reinforcement learning (RL) can significantly enhance the overall capabilities of large language models (LLMs) by optimizing their outputs against task-specific rewards (Sutton et al., 1999; Schulman et al., 2017; Ouyang et al., 2022; Rafailov et al., 2023; Trung et al., 2024; Guo et al., 2025). This paradigm has driven notable breakthroughs in complex problem-solving domains, such as mathematics and logic, where robust training signals can be derived from verifiable outcomes like ground-truth answers (Trung et al., 2024; Guo et al., 2025) or structured feedback (Bai et al., 2022b; Dubois et al., 2023). However, because these methods rely heavily on labeled datasets or mechanisms for automatic verification, their applicability remains constrained in settings where ground truth is unavailable or prohibitively expensive to collect.

To address this limitation, a growing line of research explores learning from weaker or learned supervision signals, including human preferences or model-based feedback (Ouyang et al., 2022; Bai et al., 2022a;b; Zhu et al., 2023; Rafailov et al., 2023). Concurrently, the paradigm of utilizing an "LLM-as-a-Judge" has rapidly gained traction. To date, however, this paradigm has been predominantly restricted to post-hoc evaluation—using a capable LLM to assess and rank the quality of generations (Zheng et al., 2023; Wang et al., 2023b). While these LLM evaluators correlate highly with human judgment, their potential to serve as an active, automated reward signal during RL fine-tuning remains underexplored. Existing methods that attempt to incorporate model-based feedback into training often distill it offline into static preference datasets (Bai et al., 2022b; Lee et al., 2024) or involve complex, separate reward modeling pipelines that scale poorly to the massive sampling demands of continuous RL fine-tuning (Rafailov et al., 2023; Yuan et al., 2024).

Simultaneously, as the parameter counts of state-of-the-art models continue to scale, there is a critical need to democratize advanced reasoning capabilities by distilling them into smaller, more computationally efficient models suitable for resource-constrained deployment (Hinton et al., 2015; Kim & Rush, 2016). Traditional knowledge distillation via supervised fine-tuning (SFT) often falls short; by primarily encouraging surface-level token mimicry, it fails to capture underlying reasoning processes, leading to poor generalization.

*Co-corresponding authors.

Our contributions. We bridge these gaps with a simple, scalable RL framework for label-free knowledge distillation, transforming the LLM-as-a-Judge from a static evaluator into an active reward generator. Given large volumes of unlabeled inputs, a smaller student LM generates candidate reasoning trajectories. A highly capable judge LLM then directly evaluates these candidates, producing a continuous scalar reward derived efficiently from a single next-token probability. By training the student via RL to maximize these rewards, this paradigm allows the model to actively explore the solution space and internalize the judge's latent reasoning capabilities through this lightweight feedback. On small models (125M and 350M), our RL framework achieves a 5–10 point improvement on mathematical reasoning tasks compared to using only verifiable rewards. Moreover, it can yield larger gains on out-of-domain evaluations, demonstrating improved robustness. On a larger model (6.7B), our method still provides an improvement of approximately 1.5 points, indicating that the proposed approach scales effectively to larger architectures.

2 Method

Reinforcement Learning with Verifiable Rewards (RLVR) has gained prominence following its use in DeepSeek-R1 (Guo et al., 2025) for o1-style (Learning to reason with LLMs) long chain-of-thought (CoT, Wei et al., 2022) reasoning. Yet, because RLVR's benefits scale with model size, smaller architectures experience only modest reasoning improvements (Guo et al., 2025). In this work, we demonstrate that a reinforcement learning framework guided by a carefully designed LLM-as-a-judge reward system can effectively elicit latent knowledge from a large model to enhance the reasoning capabilities of a smaller counterpart.

2.1 Problem setup

A supervised dataset consists of (question, CoT, answer) tuples denoted as (x, e, y), where the chain-of-thought (CoT) (Wei et al., 2023) e represents the intermediate reasoning process that leads to the correct answer y. The language model provides conditional probabilities of the form $\pi_\theta(a|s_t)$ where a is a token in the vocabulary, and s_t denotes the "state" at step t. The state can be interpreted as the generation context, including all tokens in the question and all tokens generated so far, based on which the LM predicts the next token. Once the token $a_{t+1} \sim \pi_\theta(a|s_t)$ is generated, it is appended to the exist context to form $s_{t+1} = [s_t, a_{t+1}]$ for generating the next token. This process is repeated until the model generates the end-of-sentence token <eos>. For the initial state, we have $s_0 = x$.

Warm-up To equip an LLM with complex reasoning capabilities, we first employ a Supervised Fine-Tuning (SFT) phase. During this process, the model is trained to maximize the log-likelihood of CoT sequences. Denote by $e = [a_1, \ldots, a_{L-1}, a_L = \text{<eos>}]$ the CoT token sequence of length L. The SFT loss is defined as $\mathcal{L}_{\text{SFT}}(\theta) = -\mathbb{E}_{e\sim D}\left[\sum_{t=1}^{L} \log(\pi_\theta(a_t|s_{t-1}))\right]$.

Reinforcement learning (RL) We can further boost the model's reasoning capability, by having the model learn from its experience using reinforcement learning (Sutton & Barto, 2018). We view $\pi_\theta(a|s_t)$ as a policy which spills out tokens, or "actions" in the RL terminology, which evolves the state s_t until reaching the terminal state (when $<$eos$>$ is generated), upon which time a *raw* reward is given based on the state trajectory. All intermediate steps have a raw reward of 0.

We adopt the Proximal Policy Gradient (PPO, Schulman et al., 2017) method as implemented by Trung et al. (2024) for RL fine-tuning of a smaller LM. Let $r(s_{t-1}, a_t, s_t)$ be the *raw* reward for generating a_t at step t; the design of raw reward is detailed in the next subsection. For stability, PPO regularizes the learned policy to stay close to the reference model π_{ref} (e.g., a model obtained by SFT warm-up), and its total reward takes into account the KL divergence between the learned model and the reference model:

$$r_{\text{total}}(s_{t-1}, a_t, s_t) = r(s_{t-1}, a_t, s_t) - \beta \cdot KL(\pi_\theta(\cdot \mid s_{t-1}), \pi_{\text{ref}}(\cdot \mid s_{t-1}))\,.$$

Additionally, PPO employs a value model $V_\phi(\cdot)$ to predict the expected reward for each state. Architecturally, V_ϕ shares most parameters with the policy π_θ by adding a value head atop the final hidden layer. We perform Temporal Difference (TD) estimation of state values:

$$\hat{A}_t = \sum_{l=0}^{L-t} (\gamma\lambda)^l \delta_{t+l}, \qquad \text{where} \quad \delta_{t'} = -V_\phi(s_{t'}) + r_{\text{total}}(s_{t'}, a_{t'+1}, s_{t'+1}) + \gamma V_\phi(s_{t'+1})$$

with the terminal state value $V_\phi(s_{L+1}) := 0$, $\gamma \in (0, 1]$ is the discount factor for rewards, and $\gamma \in [0, 1]$ is the discount factor for TD (Schulman et al., 2018). The final estimated state value for s_t is $\hat{R}_t = \hat{A}_t + V_\phi(s_t)$. The loss for step t involves two terms, one for the expected reward and the other for value estimation:

$$\mathcal{L}^t_{\text{policy}}(\theta) = -\mathbb{E}_{e\sim\pi_{\theta_{\text{old}}}}\left[\min\left(\frac{\pi_\theta(a_t \mid s_{t-1})}{\pi_{\theta_{\text{old}}}(a_t \mid s_{t-1})}\hat{A}_t,\ \text{clip}\left(\frac{\pi_\theta(a_t \mid s_{t-1})}{\pi_{\theta_{\text{old}}}(a_t \mid s_{t-1})}, 1-\epsilon, 1+\epsilon\right)\hat{A}_t\right)\right],$$

$$\mathcal{L}^t_{\text{value}}(\phi) = \frac{1}{2}\mathbb{E}_{e\sim\pi_{\theta_{\text{old}}}}\left[\max\left(\left\|V_\phi(s_t) - \hat{R}_t\right\|^2,\ \left\|\text{clip}\left(\hat{R}_t - V_\phi(s_t), \hat{A}_t - \epsilon, \hat{A}_t + \epsilon\right)\right\|^2\right)\right]$$

where $\pi_{\theta_{\text{old}}}$, $V_{\phi_{\text{old}}}$ denotes earlier models which are used for sampling the "rollout" CoT and computing $\hat{A}_t$ and $\hat{R}_t$. The final RL loss is the weighted combination:

$$\mathcal{L}_{\text{PPO}}(\theta, \phi) = \sum_{t=1}^{L}\left(\mathcal{L}^t_{\text{policy}} + \alpha\mathcal{L}^t_{\text{value}}\right)$$

where $\alpha \geq 0$ is the coefficient for the value objective.

Group Relative Policy Optimization (GRPO, Shao et al., 2024) We have implemented GRPO (see Appendix A for the formal loss definition) and benchmarked its performance against PPO (Section 4.2). Because both algorithms yield comparable results on our target tasks, and GRPO inherently demands higher computational overhead by generating multiple candidate responses per question (group size $G = 8$ in our setup), we have relied almost exclusively on PPO for the remainder of this study to maximize training efficiency.

2.2 Reward Design

2.2.1 *Verifiable Reward*

The majority of previous works have considered using binary reward or its simple extension, partial reward (Le et al., 2022; Trung et al., 2024) to reduce the effect of learning from sparse signals (Riedmiller et al., 2018; Trott et al., 2019), i.e.,

$$r(s_{t-1}, a_t, s_t) = \begin{cases} 0, & \texttt{EXTRACT}(s_L) = \emptyset \\ 0.1, & \texttt{EXTRACT}(s_L) \neq \tilde{y} \text{ and } \texttt{EXTRACT}(s_L) \neq \emptyset \\ 1, & \texttt{EXTRACT}(s_L) = \tilde{y}, \end{cases} \tag{1}$$

with $\tilde{y}$ being the ground truth answer. In a natural language CoT setup, the model receives a reward of 0 if the final answer is non-numerical (and thus cannot be extracted), 0.1 if it is numerical but incorrect, and 1 if it is completely correct. In this work, however, we focus on a programming language (PoT) setup (Gao et al., 2023) where the reasoning process is represented as a Python program, which is more rigorous and has lower perplexity than NL. Accordingly, we assign a reward of 0 if the code fails to compile or times out, 0.1 if it executes successfully but yields an incorrect answer, and a full reward of 1 when the output perfectly matches the ground truth. The loss function associated with this reward is $\mathcal{L}^{\text{VR}}_{\text{PPO}}$.

2.2.2 *LLM-as-a-judge Reward*

In this work, we focus on scenarios where unlabeled data is abundant, yet verifiable ground-truth rewards are prohibitively expensive or unavailable. To address this, we leverage a highly capable teacher LLM to dynamically generate training signals on the fly. Rather than using the teacher to generate pseudo-labels (such as complete Chain-of-Thought trajectories and final answers)—a process that is computationally heavy and prone to reasoning errors—we deploy it strictly as an evaluator. By judging the quality of student's generated reasoning, the teacher provides a nuanced "degree of correctness" that serves as a rich, continuous reward signal for RL optimization.

During RL fine-tuning, the student model generates a candidate reasoning trajectory (CoT) for a given question. This (question, CoT) pair is then concatenated with an evaluation prompt and fed to the large judge LLM to determine if the solution is correct. Because the prompt explicitly constrains the judge to answer only "Yes" or "No", we can simply perform a single-step conditional generation. By extracting the logits corresponding to the "Yes" and "No" tokens, we compute a continuous scalar reward directly from these values. This approach is an instance of the future constraint satisfaction score developed by Tu et al. (2024). The judge LLM evaluates candidates using the prompt detailed below:

> **You are a computer science teacher evaluating a student's Python solution.**
>
> **Question:** {`question`}
>
> **Python solution:** {`response`}
>
> **Ignoring the docstring, does the Python solution correctly solve the problem? Answer Yes or No only.**
>
> **Answer:** □

To compute the reward, we extract the judge LLM's specific logits for the "Yes" and "No" tokens from the output distribution at position □, calculating the score as follows:

$$s(p(\texttt{Yes}), p(\texttt{No})) = \sigma(u) \cdot \mathbb{I}_{\{\sigma(u) \geq a\}}, \qquad \text{where } u = \frac{\log p(\texttt{Yes}) - \log p(\texttt{No})}{\tau}, \tag{2}$$

$\tau \geq 0$ is a temperature parameter, $\sigma(u) = \frac{1}{1+\exp(-u)}$ is the logistic sigmoid function, and a is a threshold so that scores smaller than a will be set to 0. Throughout this work, we set $\tau = 1$ and $a = 0.35$. We emphasize that the reward depends only on LLM logits and not the plausibility of the code. Furthermore, our LLM-based reward relies on the logits from a single inference step, eliminating the need to generate complete reasoning trajectories to obtain a score. We denote the loss function associated with this reward as $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$.

2.2.3 *Pretrained Reranking Model-as-a-judge*

While reranking typically boosts *inference* by selecting best-of-k results via a binary classifier (Uesato et al., 2022), this classifier inherently outputs a correctness probability. We adapt this for *training* by repurposing a *pretrained* reranking model[1] (Trung et al., 2024). Treating its predicted probability as a continuous reward on unlabeled data integrates a second judge to diversify our RL pipeline, with the associated loss denoted $\mathcal{L}_{\text{PPO}}^{\text{Rerank}}$. However, as this reranker is task-specific, its reward may be less robust than the more task-invariant $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$.

2.2.4 *Overall training objective*

Combining the above three loss functions, we obtain the final learning objective:

$$\mathcal{L}_{\text{overall}} = \lambda \cdot \mathcal{L}_{\text{PPO}}^{\text{VR}} + \mu \cdot \mathcal{L}_{\text{PPO}}^{\text{YoN}} + \rho \cdot \mathcal{L}_{\text{PPO}}^{\text{Rerank}},$$

where the loss coefficients sum to one, forming a convex combination (i.e., $\lambda + \mu + \rho = 1$).

3 Related Work

Knowledge Distillation. Knowledge distillation (KD) transfers knowledge from a large teacher model to a smaller student model (Hinton et al., 2015). Early work in NLP extends KD to sequence generation tasks by distilling sequence-level outputs (Kim & Rush, 2016). A common approach is to minimize the Kullback–Leibler (KL) divergence between teacher and student distributions, which has been widely adopted in subsequent studies (Kim

[1] `https://huggingface.co/lqtrung1998/galactica-6.7b-ReFT-Rerank-GSM8k`

& Rush, 2016; Gu et al., 2026; Wu et al., 2025). Beyond likelihood-based objectives, Tu & Gimpel (2018); Tu et al. (2020) proposed training an inference network to match the teacher by minimizing its energy function. Recent advancements have shifted focus toward distilling complex reasoning capabilities from LLMs. For instance, Deng et al. (2023) explore a novel paradigm of *implicit* chain-of-thought distillation, transferring the explicit, token-by-token reasoning steps of a teacher directly into the vertical hidden states of a student to eliminate the need for generating intermediate text. Building on this push for extreme model compression and efficiency, recent frameworks like CODI (Shen et al., 2025) utilize self-distillation to compress natural language reasoning into a continuous latent space by aligning hidden states across tasks, successfully matching explicit reasoning performance while drastically reducing computational overhead. Most recently, research has integrated RL to further elicit and distill these complex reasoning behaviors, as seen in DeepSeek-R1 (Guo et al., 2025). However, conventional knowledge distillation via supervised fine-tuning (SFT) is limited to surface-level imitation, failing to capture underlying reasoning processes and resulting in poor generalization.

Reinforcement Learning. Reinforcement learning (RL) has become a standard approach for aligning large language models (LLMs) with desired behaviors. Early work on reinforcement learning from human feedback (RLHF) relies on reward models trained from costly human annotations (Ouyang et al., 2022; Bai et al., 2022a). To reduce this dependence, subsequent methods introduce model-based feedback, where LLMs replace human labelers (Bai et al., 2022b; Lee et al., 2024). However, these approaches typically construct static preference datasets and train separate reward models, resulting in complex and computationally expensive pipelines. To simplify training, methods such as Direct Preference Optimization (DPO) (Rafailov et al., 2023) and related techniques (Zhu et al., 2023; Yang et al., 2024) optimize policies directly from pairwise preferences, while Self-Rewarding Language Models (Yuan et al., 2024) iteratively generate their own preference data for DPO training. Nonetheless, these approaches remain limited by offline preference data. More recent work explores richer reward signals beyond binary feedback (Damani et al., 2026; Gunjal et al., 2026), and integrates RL with distillation to transfer reasoning capabilities (Guo et al., 2025). In contrast, we propose a reinforcement learning–based knowledge distillation framework that leverages multiple reward signals derived from LLM judges. Our approach produces lightweight, real-time rewards from unlabeled data, eliminating the need for offline preference datasets and ground-truth supervision while achieving strong performance on reasoning tasks.

4 Experiments

Supervised training sets. GSM8K (Cobbe et al., 2021) and SVAMP (Patel et al., 2021) are commonly used grade school level math word problem dataset in training and testing reasoning LMs. In this work, we use these two datasets for training with labels.[2]

Unsupervised datasets. To boost RL distillation performance, we apply proposed reward signals across two unlabeled datasets, specifically GSM8K-aug (Deng et al., 2024) and GSM-Plus (Li et al., 2024). While GSM-Plus typically serves as an evaluation benchmark, we ensure no ground-truth labels are revealed during the RL fine-tuning phase.[3] We specifically selected this dataset because its expert-reviewed, perturbed problems provide the necessary volume and complexity to thoroughly probe the limits of the model's reasoning capabilities.

Evaluation sets. Various challenging evaluation datasets, such as GSM-IC (Shi et al., 2023), GSM-Plus (Li et al., 2024), GSM-Symbolic (Mirzadeh et al., 2025), GSM-∞ (Zhou et al., 2025), are proposed to evaluate the robustness of language models. In this work, we focus on GSM-Plus and GSM-Symbolic. From the 10,552 total samples in GSM-Plus, we exclude 1,319 instances that lack a valid ground-truth answer (specifically, those labeled as "None").

[2]Given the non-trivial computational cost required to obtain MathQA results in Trung et al. (2024), coupled with the minimal improvement RL offers over SFT on this dataset, we chose to exclude it from our current study to focus on more computationally efficient benchmarks.

[3]We obtained explicit permission from Li et al. to use GSM-Plus for training on this project.

Dataset/Size	Train	Test
Labeled Dataset for $\mathcal{L}_{\text{PPO}}^{\text{VR}}$		
GSM8K	7,356	1,319
SVAMP	3,043	1,000
MathQA	15,250	1,605
Unlabeled Dataset for $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$ & $\mathcal{L}_{\text{PPO}}^{\text{Rerank}}$		
GSM8K-aug	385,461	—
GSM-Plus	9,233	—
Dataset for Evaluation		
GSM-Plus	—	9,233
GSM-Symbolic		
–main	—	5,000
–p1	—	5,000
–p2	—	2,500

Table 1: Dataset Statistics.

LLM	**GSM8K**		**SVAMP**	
	YoN	**LL**	**YoN**	**LL**
Qwen3-8B	88.0	81.5	93.0	79.0
Llama3.1-8B-Instruct	82.0	81.0	87.5	79.5
Gemma3-12B	85.0	49.5	87.0	64.0

Table 2: Judge Quality for Galactica-125M SFT Samples.

Dataset	**Performance**
GSM8K	87.5
SVAMP	80.5

Table 3: GSM8K-pretrained 6.7B Rerank Model Quality.

GSM-Symbolic follows a template that perturbs numeric variables in the original GSM8K dataset. To further increase the difficulty, one (p1) or two (p2) clauses are added to the questions, giving the dataset long context and making it significantly more difficult. A summary of datasets we used can be found in Table 1.

Model Architecture. We evaluate our distillation framework on Galactica-125M (Taylor et al., 2022) and CodeGen-350M (Nijkamp et al., 2023), which serve as ideal, representative small LLMs for this task. Although computational limits necessitated fine-tuning with fewer epochs and a smaller global batch size than those used by Trung et al. (2024), we still aim to reproduce their core performance for RLVR. See Appendix C.2 for hyperparameters.

4.1 Judge Quality

To achieve optimal distillation performance, we must first secure a high-quality judge for the target tasks. Therefore, we begin by comparing the performance of various large models. To do so, we sample 200 questions using the student model's SFT checkpoint, prompting the large model to evaluate both a correct (y_w) and a incorrect (y_l) answer (based on ground truth) generated by the same small model. We count the evaluation as a correct judgment if $r(x, y_w) > r(x, y_l)$. Furthermore, we compare this approach to another scoring method, denoted "LL", which returns the average token log-likelihood of the `response` portion. As shown in Table 2, Qwen3-8B performs consistently better than other models, so we adopt it as our judge in the following experiments. It is also worth noting that our scoring method outperforms the average generation token likelihood in most cases.

Following the same procedure, we next evaluate the pretrained reranking model detailed in Section 2.2.3. A judgment is considered correct if the classifier assigns a higher probability to a verifiably correct response than to an incorrect one for the same question. As shown in Table 3, the model performs exceptionally well on GSM8K, likely because it was explicitly trained on this dataset. While developing task-specific reranking models could potentially yield even stronger performance on other datasets, we leave this exploration to future work.

Table 4: Galactica-125M trained on GSM8K. Ablation of coefficients for our methods.

Coefficients			GSM8K	GSM-Plus	GSM-Symbolic			SVAMP
λ	μ	ρ			main	p1	p2	
Reported by Trung et al. (2024)								
SFT	Complete		23.70	—	—	—	—	—
1.0	–	–	29.80	—	—	—	—	—
With original GSM8K only								
SFT	Warm-up		21.83	12.49	20.24	4.88	1.80	29.50
SFT	Complete		25.25	15.14	22.88	7.40	3.64	32.30
1.0	0.0	0.0	30.48	18.38	30.96	8.42	0.48	31.30
0.5	0.5	0.0	30.63	18.96	28.32	9.64	1.40	35.90
0.0	1.0	0.0	29.19	17.95	26.42	11.70	**4.72**	37.00
GSM8K sup + GSM8K-aug unsup								
0.0	1.0	0.0	33.13	21.28	30.38	11.36	3.32	44.70
0.0	0.0	1.0	32.98	20.92	31.12	10.36	2.60	46.30
0.5	0.5	0.0	35.10	22.14	34.08	12.76	3.32	40.70
0.0	0.5	0.5	**40.71**	**25.33**	**37.74**	14.62	3.00	**50.70**
0.5	0.25	0.25	40.11	25.26	37.50	**15.70**	2.24	42.60

4.2 Results on GSM8K

We first train a supervised fine-tuning (SFT) warm-up model, which achieves an initial accuracy of 21.83%. All subsequent RL experiments are initialized from this warm-up checkpoint. Additionally, we record the performance of a complete SFT run trained to convergence; our final SFT accuracy of 25.25% on the GSM8K evaluation set is slightly stronger than the 23.70% baseline reported by Trung et al. (2024).

As shown in Table 4, we first reproduced the original PPO results, where RLVR benefits small models without extra amount of data, obtaining an accuracy of 30.48% which significantly outperforms complete SFT training (25.25%). Furthermore, using YoN reward with or without RLVR, we obtain similar performance: we achieve 30.63% for $(\lambda, \mu, \rho) = (0.5, 0.5, 0.0)$ and 29.19% for $(0.0, 1.0, 0.0)$, showing the reliability of the judge model and YoN reward.

Next, we incorporate unlabeled augmented data, where $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$ and $\mathcal{L}_{\text{PPO}}^{\text{Rerank}}$ are computed. With only unlabeled augmented data, using $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$ alone (33.13%) or $\mathcal{L}_{\text{PPO}}^{\text{Rerank}}$ (32.98%) alone outperforms $\mathcal{L}_{\text{PPO}}^{\text{VR}}$ on GSM8K only (30.48%), showing that large-scale training on unlabeled data with "noisy" rewards can surpass the performance of using limited labeled data with verifiable rewards.

In the semi-supervised setup, we compute the $\mathcal{L}_{\text{PPO}}^{\text{VR}}$ loss on the labeled data, while applying the $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$ and $\mathcal{L}_{\text{PPO}}^{\text{Rerank}}$ losses to the unlabeled data. This configuration yields a model with 40.11% accuracy using the hyperparameters $(\lambda, \mu, \rho) = (0.5, 0.25, 0.25)$, and 40.71% accuracy with the setting $(0.0, 0.5, 0.5)$. Both results represent a significant leap in performance compared to relying solely on RLVR with labeled data. See Figure 1 for learning curves.

Ablation on unsupervised data An ablation on the size of augmented data is performed in Table 5, showing that with reduced amount of unlabeled data, our model still outperforms RLVR. We then used GSM-Plus as our unlabeled dataset, and as shown in Table 5, the model performance is at 41.70%. This shows that a high-quality, difficult dataset can boost the performance of small models with less amount of data. Surprisingly, using all three loss terms enable our small model to perform better than the oracle experiment where $\mathcal{L}_{\text{PPO}}^{\text{VR}}$ is used on *labeled* GSM8K-aug, showing the efficacy of our method.

Out-of-domain evaluation Our models significantly outperform RLVR on out-of-distribution datasets—including GSM-Plus, GSM-Symbolic-main, GSM-Symbolic-p1, and SVAMP—demonstrating the robustness of our approach. On the other hand, GSM-Symbolic-p2 requires a much larger model capacity.

Table 5: Galactica-125M trained on GSM8K. Results with ablation of augmentation data.

Coefficients			GSM8K	GSM-Plus	GSM-Symbolic			SVAMP
λ	μ	ρ			main	p1	p2	
GSM8K sup + GSM8K-aug unsup								
0.5	0.25	0.25	40.11	25.26	**37.50**	**15.70**	2.24	42.60
GSM8K sup + 25% GSM8K-aug unsup								
0.5	0.25	0.25	38.36	24.61	36.42	13.74	1.16	42.30
GSM8K sup + 5% GSM8K-aug unsup								
0.5	0.25	0.25	36.54	22.85	31.96	10.24	0.96	43.10
GSM8K sup + GSM-Plus unsup								
0.5	0.5	0.0	**41.70**	**28.97**	35.52	12.78	1.12	37.90
RLVR with labeled GSM8K-aug (Oracle)								
1.0	0.0	0.0	36.01	26.64	37.20	14.58	1.96	51.00

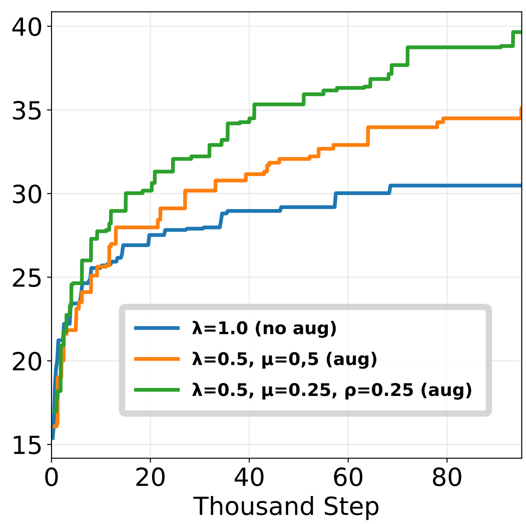


Figure 1: Evaluation Accuracy on GSM8K during RL.

GRPO To determine whether a more advanced reinforcement learning algorithm yields superior results in our framework, we evaluate the performance of GRPO. As shown in the top section of Table 6 and Figure 2, GRPO provides a more substantial performance gain. Specifically, under the configuration $(\lambda, \mu, \rho) = (0.5, 0.5, 0.0)$, GRPO achieves an accuracy of 37.30%, outperforming the 35.10% accuracy obtained by PPO under identical settings.

Large model (Galactica-6.7B) It is natural to investigate whether our proposed reward framework benefits larger architectures. As detailed in Table 6 (bottom section), we demonstrate that our approach can effectively boost the RL performance of a student model using feedback from a judge of comparable scale. We initialize our experiments from the 6.7B Galactica checkpoint provided by Trung et al. (2024) on both $\mathcal{L}_{\text{PPO}}^{\text{VR}}$ and $\mathcal{L}_{\text{PPO}}^{\text{VR}} + \mathcal{L}_{\text{PPO}}^{\text{YoN}}$ losses[4]. Although the baseline RLVR performance is already saturated at this scale, incorporating the combined reward yields a notable gain, improving accuracy from 69.07% to 70.43% on the GSM8K evaluation set. Applying majority voting (Wang et al., 2023a) further elevates this result to 71.95%. Ultimately, these findings illustrate that effective distillation can be driven not just by a disparity in model size, but also by scaling the volume of training data.

CodeGen-350M. In Table 7, we show that similar performance gains are observed using another small model, CodeGen-350M. When we mix labeled data for $\mathcal{L}_{\text{PPO}}^{\text{VR}}$ and unlabeled data for $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$ and $\mathcal{L}_{\text{PPO}}^{\text{Rerank}}$, we obtain a model with 31.31% accuracy on GSM8K test set,

[4] Using all three losses $\mathcal{L}_{\text{PPO}}^{\text{VR}} + \mathcal{L}_{\text{PPO}}^{\text{YoN}} + \mathcal{L}_{\text{PPO}}^{\text{Rerank}}$ on a 6.7B model would necessitate ZeRO-3 offloading, rendering the training process prohibitively slow; therefore, we do not pursue it here.

Table 6: Galactica trained on GSM8K. Small model with GRPO and large model with PPO.

Coefficients			GSM8K	GSM-Plus	GSM-Symbolic			SVAMP
λ	μ	ρ			main	p1	p2	
Galactica-125M (GRPO)								
1.0	0.0	0.0	30.33	18.00	26.30	7.98	1.36	28.50
0.5	0.5	0.0	**37.30**	**23.71**	**35.36**	**11.18**	2.20	**45.20**
Galactica-6.7B (PPO)								
Reported by Trung et al. (2024)								
1.0	0.0	0.0	68.91	—	—	—	—	—
Continuing from the above checkpoint								
1.0	0.0	0.0	69.07	51.97	64.24	54.82	23.36	69.40
+ voting			71.72	53.28	67.42	**57.94**	27.00	70.80
0.5	0.5	0.0	70.43	53.35	66.32	53.32	25.72	74.20
+ voting			**71.95**	**55.52**	**68.12**	57.28	**28.36**	**74.90**

Table 7: CodeGen-350M trained on GSM8K. $\times$ indicates training collapse.

Coefficients			GSM8K	GSM-Plus	GSM-Symbolic			SVAMP
λ	μ	ρ			main	p1	p2	
Reported by Trung et al. (2024)								
SFT	Complete		20.40	—	—	—	—	—
1.0	–	–	28.40	—	—	—	—	—
Original GSM8K only								
SFT	Warm-up		22.37	13.14	23.14	8.44	2.84	27.20
SFT	Complete		22.97	14.07	23.40	8.34	2.96	28.20
1.0	0.0	0.0	26.91	16.35	29.90	10.54	4.16	34.30
0.5	0.5	0.0	27.52	16.59	30.06	9.14	3.04	38.20
0.0	1.0	0.0	$\times$	—	—	—	—	—
GSM8K sup + GSM8K-aug unsup								
0.0	1.0	0.0	$\times$	—	—	—	—	—
0.5	0.5	0.0	31.01	19.11	**33.86**	11.54	**4.72**	**38.50**
0.5	0.25	0.25	31.31	19.63	30.38	10.74	3.56	35.90
GSM8K sup + GSM-Plus unsup								
0.5	0.5	0.0	**35.63**	**22.98**	33.12	**12.08**	2.08	32.80

showing a substantial increase from 26.91%, which uses only RLVR on labeled data. Using $\mathcal{L}_{\text{PPO}}^{\text{VR}} + \mathcal{L}_{\text{PPO}}^{\text{YoN}}$ already gives us a performance of 31.01%, so the rerank reward does not seem to contribute as much to distillation as it does to Galatica-125M. Our best result 35.63% is achieved by semi-supervised training on GSM-Plus with $\mathcal{L}_{\text{PPO}}^{\text{YoN}}$. We note that reward signals on only unlabeled data may lead to the reward hacking phenomenon, causing model collapse. However, RLVR stabilizes training.

4.3 Results on SVAMP

As shown in Table 8, our method improves small models significantly on SVAMP as well. For Galactica-125M, we show that with augmented data, we can improve the small model performance from that with RLVR (40.1%) to 47.0% with all three loss terms. And for CodeGen-350M, performance is improved from 41.4% to 46.6%. We observe that models trained on GSM8K generalize effectively to SVAMP, whereas models trained on SVAMP perform poorly on the GSM8K evaluation sets (results not shown). This asymmetry indicates that the similarity between the labeled training set and the target domain critically determines the model's ultimate robustness and generalization. Model collapse is noticed when $\lambda = 0$; we discuss this phenomenon and mitigation strategies in Appendix B.3.

Table 8: Small models trained and evaluated on SVAMP.

Galactica-125M

Coefficients λ	μ	ρ	SVAMP
Reported by Trung et al. (2024)			
SFT	Complete		35.60
1.0	–	–	39.40
Original SVAMP only			
SFT	Warm-up		36.90
SFT	Complete		37.40
1.0	0.0	0.0	40.10
0.5	0.5	0.0	40.10
0.0	1.0	0.0	39.90
GSM8K sup + GSM8K-aug unsup			
0.0	1.0	0.0	44.50
0.5	0.5	0.0	**47.00**
0.5	0.25	0.25	44.30

CodeGen-350M

Coefficients λ	μ	ρ	SVAMP
Reported by Trung et al. (2024)			
SFT	Complete		34.40
1.0	–	–	39.30
Original SVAMP only			
SFT	Warm-up		35.60
SFT	Complete		35.80
1.0	0.0	0.0	41.40
GSM8K sup + GSM8K-aug unsup			
0.0	1.0	0.0	×
0.5	0.5	0.0	41.80
0.5	0.25	0.25	**46.60**

5 Conclusion and Future Work

In this work, we propose an LLM fine-tuning framework that delivers consistent and substantial performance gains across challenging mathematical reasoning benchmarks. Our findings highlight that efficient LLM-based evaluators can serve as powerful training signals for RL fine-tuning, significantly enhancing the reasoning capabilities of smaller models while reducing dependence on ground-truth supervision. For future work, we plan to enhance our distillation framework by integrating more advanced variants of policy gradient algorithms (Yu et al., 2025). Additionally, we aim to extend our methodology to the training and fine-tuning of emerging diffusion language models (Lou et al., 2024; Ye et al., 2024; Nie et al., 2025; Zekri & Boullé, 2025; Zhu et al., 2025).

References

Yuntao Bai, Andy Jones, Kamal Ndousse, Amanda Askell, Anna Chen, Nova DasSarma, Dawn Drain, Stanislav Fort, Deep Ganguli, Tom Henighan, Nicholas Joseph, Saurav Kadavath, Jackson Kernion, Tom Conerly, Sheer El-Showk, Nelson Elhage, Zac Hatfield-Dodds, Danny Hernandez, Tristan Hume, Scott Johnston, Shauna Kravec, Liane Lovitt, Neel Nanda, Catherine Olsson, Dario Amodei, Tom Brown, Jack Clark, Sam McCandlish, Chris Olah, Ben Mann, and Jared Kaplan. Training a helpful and harmless assistant with reinforcement learning from human feedback, 2022a. URL https://arxiv.org/abs/2204.05862. arXiv 2204.05862 [cs.CL].

Yuntao Bai, Saurav Kadavath, Sandipan Kundu, Amanda Askell, Jackson Kernion, Andy Jones, Anna Chen, Anna Goldie, Azalia Mirhoseini, Cameron McKinnon, Carol Chen, Catherine Olsson, Christopher Olah, Danny Hernandez, Dawn Drain, Deep Ganguli, Dustin Li, Eli Tran-Johnson, Ethan Perez, Jamie Kerr, Jared Mueller, Jeffrey Ladish, Joshua Landau, Kamal Ndousse, Kamile Lukosuite, Liane Lovitt, Michael Sellitto, Nelson Elhage, Nicholas Schiefer, Noemi Mercado, Nova DasSarma, Robert Lasenby, Robin Larson, Sam Ringer, Scott Johnston, Shauna Kravec, Sheer El Showk, Stanislav Fort, Tamera Lanham, Timothy Telleen-Lawton, Tom Conerly, Tom Henighan, Tristan Hume, Samuel R. Bowman, Zac Hatfield-Dodds, Ben Mann, Dario Amodei, Nicholas Joseph, Sam McCandlish, Tom Brown, and Jared Kaplan. Constitutional AI: Harmlessness from AI feedback, 2022b. URL https://arxiv.org/abs/2212.08073. arXiv 2212.08073 [cs.CL].

Karl Cobbe, Vineet Kosaraju, Mohammad Bavarian, Mark Chen, Heewoo Jun, Lukasz Kaiser, Matthias Plappert, Jerry Tworek, Jacob Hilton, Reiichiro Nakano, Christopher Hesse, and John Schulman. Training verifiers to solve math word problems, 2021. URL https://arxiv.org/abs/2110.14168.

Mehul Damani, Isha Puri, Stewart Slocum, Idan Shenfeld, Leshem Choshen, Yoon Kim, and Jacob Andreas. Beyond binary rewards: Training LMs to reason about their uncertainty. In *The Fourteenth International Conference on Learning Representations*, 2026. URL https://openreview.net/forum?id=ASQ649zdHm.

Yuntian Deng, Kiran Prasad, Roland Fernandez, Paul Smolensky, Vishrav Chaudhary, and Stuart Shieber. Implicit chain of thought reasoning via knowledge distillation, 2023. URL https://arxiv.org/abs/2311.01460.

Yuntian Deng, Kiran Prasad, Roland Fernandez, Paul Smolensky, Vishrav Chaudhary, and Stuart Shieber. Implicit chain of thought reasoning via knowledge distillation. In *The Twelfth International Conference on Learning Representations*, 2024.

Yann Dubois, Xuechen Li, Rohan Taori, Tianyi Zhang, Ishaan Gulrajani, Jimmy Ba, Carlos Guestrin, Percy Liang, and Tatsunori B. Hashimoto. AlpacaFarm: A simulation framework for methods that learn from human feedback. In *Advances in Neural Information Processing Systems*, 2023.

Luyu Gao, Aman Madaan, Shuyan Zhou, Uri Alon, Pengfei Liu, Yiming Yang, Jamie Callan, and Graham Neubig. PAL: Program-aided language models. In *International Conference on Machine Learning*, 2023.

Yuxian Gu, Li Dong, Furu Wei, and Minlie Huang. Minillm: On-policy distillation of large language models, 2026. URL https://arxiv.org/abs/2306.08543.

Anisha Gunjal, Anthony Wang, Elaine Lau, Vaskar Nath, Yunzhong He, Bing Liu, and Sean Hendryx. Rubrics as rewards: Reinforcement learning beyond verifiable domains. In *The Fourteenth International Conference on Learning Representations*, 2026.

Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, and Peiyi Wang. DeepSeek-R1: Incentivizes reasoning in LLMs through reinforcement learning. *Nature*, 645(8081): 633–638, September 2025. ISSN 1476-4687. doi: 10.1038/s41586-025-09422-z. URL http://dx.doi.org/10.1038/s41586-025-09422-z.

Geoffrey Hinton, Oriol Vinyals, and Jeff Dean. Distilling the knowledge in a neural network, 2015. URL https://arxiv.org/abs/1503.02531.

Yoon Kim and Alexander M. Rush. Sequence-level knowledge distillation. In Jian Su, Kevin Duh, and Xavier Carreras (eds.), *Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing*, pp. 1317–1327, Austin, Texas, November 2016. Association for Computational Linguistics. doi: 10.18653/v1/D16-1139. URL https://aclanthology.org/D16-1139/.

Hung Le, Yue Wang, Akhilesh Deepak Gotmare, Silvio Savarese, and Steven Chu Hong Hoi. Coderl: Mastering code generation through pretrained models and deep reinforcement learning. In S. Koyejo, S. Mohamed, A. Agarwal, D. Belgrave, K. Cho, and A. Oh (eds.), *Advances in Neural Information Processing Systems*, volume 35, pp. 21314–21328. Curran Associates, Inc., 2022. URL https://proceedings.neurips.cc/paper_files/paper/2022/file/8636419dea1aa9fbd25fc4248e702da4-Paper-Conference.pdf.

Learning to reason with LLMs, 2024. URL https://openai.com/index/learning-to-reason-with-llms/.

Harrison Lee, Samrat Phatale, Hassan Mansoor, Thomas Mesnard, Johan Ferret, Kellie Lu, Colton Bishop, Ethan Hall, Victor Carbune, Abhinav Rastogi, and Sushant Prakash. RLAIF vs. RLHF: Scaling reinforcement learning from human feedback with AI feedback. In *International Conference on Machine Learning*, 2024.

Qintong Li, Leyang Cui, Xueliang Zhao, Lingpeng Kong, and Wei Bi. GSM-plus: A comprehensive benchmark for evaluating the robustness of LLMs as mathematical problem solvers. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar (eds.), *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, pp. 2961–2984, Bangkok, Thailand, August 2024. Association for Computational Linguistics.

doi: 10.18653/v1/2024.acl-long.163. URL https://aclanthology.org/2024.acl-long.163/.

Aaron Lou, Chenlin Meng, and Stefano Ermon. Discrete diffusion modeling by estimating the ratios of the data distribution. In *International Conference on Machine Learning*, 2024.

Seyed Iman Mirzadeh, Keivan Alizadeh, Hooman Shahrokhi, Oncel Tuzel, Samy Bengio, and Mehrdad Farajtabar. Gsm-symbolic: Understanding the limitations of mathematical reasoning in large language models. In *The Thirteenth International Conference on Learning Representations*, 2025.

Shen Nie, Fengqi Zhu, Zebin You, Xiaolu Zhang, Jingyang Ou, Jun Hu, Jun Zhou, Yankai Lin, Ji-Rong Wen, and Chongxuan Li. Large language diffusion models. *arXiv preprint arXiv:2502.09992*, 2025.

Erik Nijkamp, Bo Pang, Hiroaki Hayashi, Lifu Tu, Huan Wang, Yingbo Zhou, Silvio Savarese, and Caiming Xiong. Codegen: An open large language model for code with multi-turn program synthesis. In *The Eleventh International Conference on Learning Representations*, 2023.

Long Ouyang, Jeffrey Wu, Xu Jiang, Diogo Almeida, Carroll Wainwright, Pamela Mishkin, Chong Zhang, Sandhini Agarwal, Katarina Slama, Alex Ray, et al. Training language models to follow instructions with human feedback. *Advances in Neural Information Processing Systems*, 35:27730–27744, 2022.

Arkil Patel, Satwik Bhattamishra, and Navin Goyal. Are NLP models really able to solve simple math word problems? In Kristina Toutanova, Anna Rumshisky, Luke Zettlemoyer, Dilek Hakkani-Tur, Iz Beltagy, Steven Bethard, Ryan Cotterell, Tanmoy Chakraborty, and Yichao Zhou (eds.), *Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*, pp. 2080–2094, Online, June 2021. Association for Computational Linguistics. doi: 10.18653/v1/2021.naacl-main.168. URL https://aclanthology.org/2021.naacl-main.168/.

Rafael Rafailov, Archit Sharma, Eric Mitchell, Stefano Ermon, Christopher D Manning, and Chelsea Finn. Direct preference optimization: Your language model is secretly a reward model. *arXiv preprint arXiv:2305.18290*, 2023.

Martin Riedmiller, Roland Hafner, Thomas Lampe, Michael Neunert, Jonas Degrave, Tom van de Wiele, Vlad Mnih, Nicolas Heess, and Jost Tobias Springenberg. Learning by playing solving sparse reward tasks from scratch. In Jennifer Dy and Andreas Krause (eds.), *Proceedings of the 35th International Conference on Machine Learning*, volume 80 of *Proceedings of Machine Learning Research*, pp. 4344–4353. PMLR, 10–15 Jul 2018. URL https://proceedings.mlr.press/v80/riedmiller18a.html.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. Proximal policy optimization algorithms, 2017. URL https://arxiv.org/abs/1707.06347.

John Schulman, Philipp Moritz, Sergey Levine, Michael Jordan, and Pieter Abbeel. High-dimensional continuous control using generalized advantage estimation, 2018. URL https://arxiv.org/abs/1506.02438.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Xiao Bi, Haowei Zhang, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. DeepSeekMath: Pushing the limits of mathematical reasoning in open language models, 2024. URL https://arxiv.org/abs/2402.03300.

Zhenyi Shen, Hanqi Yan, Linhai Zhang, Zhanghao Hu, Yali Du, and Yulan He. CODI: Compressing chain-of-thought into continuous space via self-distillation. In Christos Christodoulopoulos, Tanmoy Chakraborty, Carolyn Rose, and Violet Peng (eds.), *Proceedings of the 2025 Conference on Empirical Methods in Natural Language Processing*, pp. 677–693, Suzhou, China, November 2025. Association for Computational Linguistics. ISBN 979-8-89176-332-6. doi: 10.18653/v1/2025.emnlp-main.36. URL https://aclanthology.org/2025.emnlp-main.36/.

Freda Shi, Xinyun Chen, Kanishka Misra, Nathan Scales, David Dohan, Ed H. Chi, Nathanael Schärli, and Denny Zhou. Large language models can be easily distracted by irrelevant context. In Andreas Krause, Emma Brunskill, Kyunghyun Cho, Barbara Engelhardt, Sivan Sabato, and Jonathan Scarlett (eds.), *Proceedings of the 40th International Conference on Machine Learning*, volume 202 of *Proceedings of Machine Learning Research*, pp. 31210–31227. PMLR, 23–29 Jul 2023. URL https://proceedings.mlr.press/v202/shi23a.html.

Joar Skalse, Nikolaus H. R. Howe, Dmitrii Krasheninnikov, and David Krueger. Defining and characterizing reward hacking, 2025. URL https://arxiv.org/abs/2209.13085.

Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. MIT Press, Cambridge, MA, 2nd edition, 2018.

Richard S Sutton, David McAllester, Satinder Singh, and Yishay Mansour. Policy gradient methods for reinforcement learning with function approximation. In *Advances in Neural Information Processing Systems*, volume 12, 1999.

Ross Taylor, Marcin Kardas, Guillem Cucurull, Thomas Scialom, Anthony Hartshorn, Elvis Saravia, Andrew Poulton, Viktor Kerkez, and Robert Stojnic. Galactica: A large language model for science, 2022. URL https://arxiv.org/abs/2211.09085.

Alexander Trott, Stephan Zheng, Caiming Xiong, and Richard Socher. Keeping your distance: Solving sparse reward tasks using self-balancing shaped rewards. In *Advances in Neural Information Processing Systems*, 2019.

Luong Trung, Xinbo Zhang, Zhanming Jie, Peng Sun, Xiaoran Jin, and Hang Li. ReFT: Reasoning with reinforced fine-tuning. In Lun-Wei Ku, Andre Martins, and Vivek Srikumar (eds.), *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, August 2024.

Lifu Tu and Kevin Gimpel. Learning approximate inference networks for structured prediction. In *Proceedings of International Conference on Learning Representations (ICLR)*, 2018.

Lifu Tu, Richard Yuanzhe Pang, Sam Wiseman, and Kevin Gimpel. ENGINE: Energy-based inference networks for non-autoregressive machine translation. In Dan Jurafsky, Joyce Chai, Natalie Schluter, and Joel Tetreault (eds.), *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics*, pp. 2819–2826, Online, July 2020. Association for Computational Linguistics. doi: 10.18653/v1/2020.acl-main.251. URL https://aclanthology.org/2020.acl-main.251/.

Lifu Tu, Semih Yavuz, Jin Qu, Jiacheng Xu, Rui Meng, Caiming Xiong, and Yingbo Zhou. Unlocking anticipatory text generation: A constrained approach for large language models decoding. In Yaser Al-Onaizan, Mohit Bansal, and Yun-Nung Chen (eds.), *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing*, pp. 15532–15548, Miami, Florida, USA, November 2024. Association for Computational Linguistics. doi: 10.18653/v1/2024.emnlp-main.870. URL https://aclanthology.org/2024.emnlp-main.870/.

Masatoshi Uehara, Yulai Zhao, Kevin Black, Ehsan Hajiramezanali, Gabriele Scalia, Nathaniel Lee Diamant, Alex M Tseng, Tommaso Biancalani, and Sergey Levine. Fine-tuning of continuous-time diffusion models as entropy-regularized control, 2024. URL https://arxiv.org/abs/2402.15194.

Jonathan Uesato, Nate Kushman, Ramana Kumar, Francis Song, Noah Siegel, Lisa Wang, Antonia Creswell, Geoffrey Irving, and Irina Higgins. Solving math word problems with process- and outcome-based feedback, 2022. URL https://arxiv.org/abs/2211.14275.

Xuezhi Wang, Jason Wei, Dale Schuurmans, Quoc V Le, Ed H. Chi, Sharan Narang, Aakanksha Chowdhery, and Denny Zhou. Self-consistency improves chain of thought reasoning in language models. In *The Eleventh International Conference on Learning Representations*, 2023a.

Yidong Wang, Zhuohao Yu, Zhengran Zeng, Linyi Yang, Cunxiang Wang, et al. PandaLM: An automatic evaluation benchmark for llm instruction tuning optimization. *arXiv preprint arXiv:2306.05087*, 2023b.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, brian ichter, Fei Xia, Ed H. Chi, Quoc V Le, and Denny Zhou. Chain of thought prompting elicits reasoning in large language models. In *Advances in Neural Information Processing Systems*, 2022.

Jason Wei, Xuezhi Wang, Dale Schuurmans, Maarten Bosma, Brian Ichter, Fei Xia, Ed Chi, Quoc Le, and Denny Zhou. Chain-of-thought prompting elicits reasoning in large language models, 2023. URL https://arxiv.org/abs/2201.11903.

Taiqiang Wu, Chaofan Tao, Jiahao Wang, Runming Yang, Zhe Zhao, and Ngai Wong. Rethinking Kullback-Leibler divergence in knowledge distillation for large language models. In Owen Rambow, Leo Wanner, Marianna Apidianaki, Hend Al-Khalifa, Barbara Di Eugenio, and Steven Schockaert (eds.), *Proceedings of the 31st International Conference on Computational Linguistics*, pp. 5737–5755, Abu Dhabi, UAE, January 2025. Association for Computational Linguistics. URL https://aclanthology.org/2025.coling-main.383/.

Kevin Yang, Dan Klein, Asli Celikyilmaz, Nanyun Peng, and Yuandong Tian. RLCD: Reinforcement learning from contrastive distillation for LM alignment. In *The Twelfth International Conference on Learning Representations*, 2024. URL https://openreview.net/forum?id=v3XXtxWKi6.

Jiacheng Ye, Shansan Gong, Liheng Chen, Lin Zheng, Jiahui Gao, Han Shi, Chuan Wu, Zhenguo Li, Wei Bi, and Lingpeng Kong. Diffusion of thoughts: Chain-of-thought reasoning in diffusion language models. In *Advances in Neural Information Processing Systems*, 2024.

Qiying Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan, Xiaochen Zuo, Yu Yue, Weinan Dai, Tiantian Fan, Gaohong Liu, Lingjun Liu, Xin Liu, Haibin Lin, Zhiqi Lin, Bole Ma, Guangming Sheng, Yuxuan Tong, Chi Zhang, Mofan Zhang, Wang Zhang, Hang Zhu, Jinhua Zhu, Jiaze Chen, Jiangjie Chen, Chengyi Wang, Hongli Yu, Yuxuan Song, Xiangpeng Wei, Hao Zhou, Jingjing Liu, Wei-Ying Ma, Ya-Qin Zhang, Lin Yan, Mu Qiao, Yonghui Wu, and Mingxuan Wang. DAPO: An open-source llm reinforcement learning system at scale, 2025. URL https://arxiv.org/abs/2503.14476.

Weizhe Yuan, Richard Yuanzhe Pang, Kyunghyun Cho, Xian Li, Sainbayar Sukhbaatar, Jing Xu, and Jason Weston. Self-rewarding language models. In *International Conference on Machine Learning*, 2024.

Oussama Zekri and Nicolas Boullé. Fine-tuning discrete diffusion models with policy gradient methods. In *Advances in Neural Information Processing Systems*, 2025.

Lianmin Zheng, Wei-Lin Chiang, Ying Sheng Ying, Shiyang Hao, Sean Macavaney, et al. Judging llm-as-a-judge with mt-bench and chatbot arena. In *Advances in Neural Information Processing Systems*, volume 36, 2023.

Yang Zhou, Hongyi Liu, Zhuoming Chen, Yuandong Tian, and Beidi Chen. Gsm-∞: How do your llms behave over infinitely increasing reasoning complexity and context length? In *Forty-second International Conference on Machine Learning*, 2025.

Banghua Zhu, Jiantao Jiao, and Michael I Jordan. Principled reinforcement learning with human feedback from pairwise or k-wise comparisons. *arXiv preprint arXiv:2301.11270*, 2023.

Fengqi Zhu, Rongzhen Wang, Shen Nie, Xiaolu Zhang, Chunwei Wu, Jun Hu, Jun Zhou, Jianfei Chen, Yankai Lin, Ji-Rong Wen, and Chongxuan Li. Llada 1.5: Variance-reduced preference optimization for large language diffusion models, 2025. URL https://arxiv.org/abs/2505.19223.

A Group Relative Policy Optimization (GRPO)

Due to the success of DeepSeek-R1 (Guo et al., 2025), GRPO has become currently the most popular RL algorithm for reasoning tasks, and one may wonder how it compares with PPO on our tasks. Here we briefly review the algorithm.

A key architectural divergence lies in the absence of a value module in GRPO. Unlike PPO, which relies on a critic to estimate the advantage and mitigate the variance of the policy gradient, GRPO eliminates this requirement. Instead, GRPO samples a group of CoTs for the same question, denoted as $\{e^1, \ldots, e^G\}$ where $e^i = [a^i_1, \ldots, a^i_{|e^i|}]$, using the old policy model $\pi_{\theta_{\text{old}}}$. They receive the raw rewards $\{r^1, \ldots, r^G\}$ at their corresponding terminal state. Outcome supervision provided the normalized reward that is shared by all tokens within each CoT:

$$\hat{A}^i = \frac{r^i - \text{mean}(r)}{\text{std}(r)}.$$

GRPO optimizes the following objective

$$\mathcal{L}_{\text{GRPO}}(\theta) = \mathbb{E}_{\{e^i\}_{i=1}^G \sim \pi_{\theta_{\text{old}}}} \frac{1}{G} \sum_{i=1}^{G} \frac{1}{|e^i|} \times$$

$$\sum_{t=1}^{|e^i|} \left\{ \min \left[\frac{\pi_\theta(a^i_t \mid s^i_{t-1})}{\pi_{\theta_{\text{old}}}(a^i_t \mid s^i_{t-1})} \hat{A}^i, \text{clip} \left(\frac{\pi_\theta(a^i_t \mid s^i_{t-1})}{\pi_{\theta_{\text{old}}}(a^i_t \mid s^i_{t-1})}, 1 - \varepsilon, 1 + \varepsilon \right) \hat{A}^i \right] - \beta \mathbb{D}_{\text{KL}} \left[\pi_\theta \| \pi_{\text{ref}} \right] \right\}.$$

A.1 Accuracy Curve

The delta with GRPO is larger than that of PPO as shown in Figure 2, where aug means using GSM8K-aug as unlabeled data.

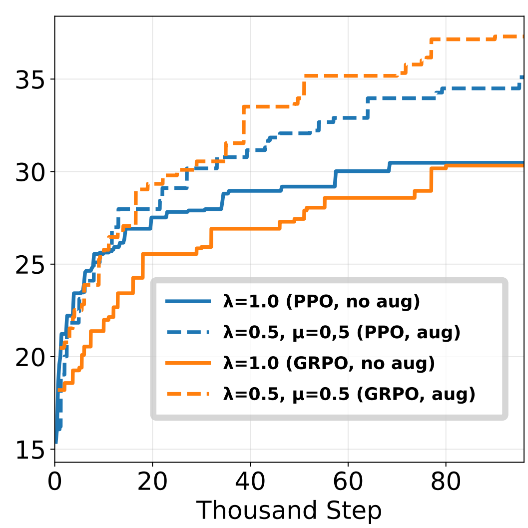
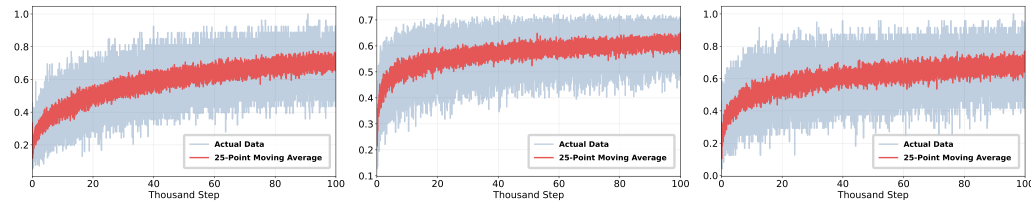


Figure 2: PPO vs GRPO

B Additional Results

B.1 Reward Curves

These three curves in Figure 3 show that the reward increases and plateaus later. It corresponds to our setup where $\lambda = 0.5, \mu = 0.25, \rho = 0.25$ using GSM8K-aug.



(a) Verifiable Reward Mean (b) Judge Reward Mean (c) Rerank Model Reward Mean

Figure 3: All three types of our active training reward on Galactica-125M.

B.2 Natural language tasks.

For natural lanuage training, we use the following prompt:

> **You are a grade school math teacher trying to evaluate the student's response.**
>
> **Question:** {`question`}
>
> **Response:** {`response`}
>
> **Is the response correct? Answer Yes or No only.**
>
> **Answer:** □

Table 9: Galactica-125M trained on GSM8K-NL. First row is an SFT run.

Coefficients			GSM8K	GSM-Plus	GSM-Symbolic			SVAMP
λ	μ	ρ			main	p1	p2	
Reference GSM8K-NL								
–	–	–	8.49	4.17	3.42	1.16	0.32	7.40
1.0	0.0	0.0	10.31	5.04	1.20	1.20	1.24	9.10
With GSM8K-aug								
0.5	0.5	0.0	**10.46**	**5.22**	**4.18**	**1.42**	**0.92**	**11.10**

As shown in Table 9, because small models benefit more from PoT, which uses highly structured language, than from NL CoT, we use the former for all of our experiments.

B.3 Failure Modes

Reward hacking or reward collapse has been reported in existing RL literature (Uehara et al., 2024; Skalse et al., 2025). It is a phenomenon where the nominal (or proxy) reward values are high (e.g., the aesthetic score of a generated image, the plausibility of generated code snippet), while the generation itself does not make sense. In our experiments, this often occur when $\lambda = 0$. For example, as seen in Figure 5, the hacked response does not generate any code (or in some cases, generates a number with no reasoning whatsoever and returns directly), but the reward is much higher than the incorrect response with reasonable code. As we further demonstrate in Figure 4, the transition from stable state to failure state happens quickly. We present some intuitive mitigation strategies as follows, but more rigorous treatments are left to future explorations.

Mitigation Strategies Occasionally, reward hacking persists in the unsupervised dataset even when the supervised dataset generates reasonable signals. This may lead to training collapse or drastically slow down training. To alleviate this, we (1) instruct the small LMs explicitly by the end of the docstring "`Please provide full reasoning before computing result; do not answer with result only.`" and (2) apply a mask, so that if generated code snippet is less than 4 lines, we assign a judge reward of 0. This modification is only

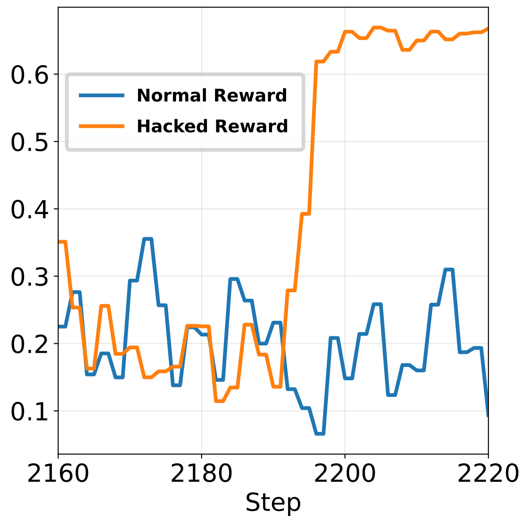


Figure 4: Judge Mean Reward Curve

Question: each pack of dvds costs 76 dollars. if there is a discount of 25 dollars on each pack how much do you have to pay to buy each pack?

Correct Response

```
def solution():
    """each pack of dvds costs 76...
    ...how much do you have to pay?"""
    cost_per_pack = 76
    discount_cost = 25
    total_cost = cost_per_pack
                 - discount_cost
    return total_cost
```

Reward: 0.728

Incorrect Response

```
def solution():
    """each pack of dvds costs 76...
    ...how much do you have to pay?"""
    cost_per_pack = 76
    discount_amount = 25
    total_cost = cost_per_pack
                 * discount_amount
    return total_cost
```

Reward: 0.303 (clipped to 0)

Hacked Response

```
def solution():
    """each pack of dvds costs 76...
    ...how much do you have to pay?"""
```

Reward: 0.635

Figure 5: Example completions with YoN reward scores. Notice how the hacked response does not generate anything given the declaration and docstring, yet obtaining a good score.

applied to GSM8K sup+GSM-Plus unsup using CodeGen in Table 7. However, these patches may not solve the issue since occasionally our modifications led to consistent zero rewards and henceforth model collapse. They also cause performance degradation in runs that do not observe reward hacking.

The instance of reward hacking reported in Trung et al. (2024) on MathQA-Multiple Choice dataset, where the model reports the correct answer option albeit having the wrong reasoning, is different from our encounter, where the model gives no or very short response without any reasoning. Nevertheless, both scenarios achieve high rewards with low quality reasoning. More future work on new regularization methods is therefore necessary.

C Implementation Details

All experiments are performed on A100 GPUs and H100 GPUs.

C.1 Data Processing

Sampling. We created 100 completions/responses from 200 randomly selected test questions, using the SFT warm-up model and same seed for each test set. We select one correct completion and one incorrect completion and use the LLM to score the correct completion and the incorrect completion.

Training. Grade school-level math reasoning datasets GSM8K and SVAMP are directly obtained from the ReFT repository (Trung et al., 2024). The augmented version of GSM8K (GSM8K-aug) is first created in Deng et al. (2024), and we use the processed version on Huggingface[5]. For ablation on the amount of augmented data, we randomly select 5% (19,273) and 25% (96,365) of all GSM8K-aug.

C.2 Hyperparameters

Training steps. Because we have two dataloaders for distillation experiments, we do not count epochs and instead count optimizer update steps. Furthermore, since GSM8K-aug has a large amount of data, running one epoch on unlabeled data will take at least six days and it does not make sense for us to track number of epochs. For Galactica-125M[6] experiments trained on GSM8K, we trained for 100,000 optimizer steps. For CodeGen-350M[7] experiments trained on GSM8K, we trained for 50,000 steps. For Galactica-6.7B experiments continuing from the checkpoint, we ran 15,000 optimizer steps. For all experiments on SVAMP, we ran for 50,000 steps.

SFT. To warm-up a model, we train the model for 10 epochs using 24 global batch size. For complete SFT runs, we train the model for 40 epoch using 24 global batch size. The learning rate is 2e-5 in both scenarios.

RL-based distillation. We train all small models based on warm-up checkpoints, using global batch size of 24. The PPO epoch is set to 2. The value function coefficients α for Galactica-125M and CodeGen-350M are set to 5 and 0.1, respectively. The KL coefficient β is set to 0.01 for all PoT tasks.

GRPO. The global batch size is 16, and we use 8 hypotheses ($G = 8$) per question to calculate the advantage. The other settings are identical to those of PPO.

Large Model Experiment. The global batch size is 16, and we use 100 hypotheses per question for majority voting.

More on the overall objective. While the reward functions can be simply summed up, leading to only one PPO loss, we use three losses here because one dataloader is used for labeled data, and the other unlabeled data. To make fair comparisons across methods, the recorded steps are optimizer update steps instead of those for a certain dataloader.

[5] https://huggingface.co/datasets/whynlp/gsm8k-aug

[6] https://huggingface.co/facebook/galactica-125m

[7] https://huggingface.co/Salesforce/codegen-350M-mono